Viscous/Inviscid Analysis of Curved Sub- or Supersonic Wall Jets

S. M. Dash,* R. A. Beddini,† D. E. Wolf,‡ and N. Sinha‡
Science Applications, Inc., Princeton, New Jersey

Abstract

THE features of a viscous/inviscid curved wall jet model, under development for application to the analysis of circulation-controlled airfoil flowfields, are described. The model solves the surface-oriented parabolized Navier-Stokes equations using pressure-split methodology for subsonic wall jets and shock-capturing methodology for supersonic wall jets. A hybrid two-layer turbulence model is employed that combines a damped VanDriest inner layer formulation with a curvature-corrected, two-equation $k\epsilon$ model outer region formulation. Procedures for performing the strongly interactive coupling of the wall jet solution with an external potential flow solver are discussed. Preliminary calculations for simple wall jets are shown to compare quite favorably with both mean flow and turbulent measurements.

Contents

Wall jets play a fundamental role in the overall aerodynamics of V/STOL fixed and rotary wing aircraft. These wall jets are generally immersed in a complex subsonic/transonic flow environment, are often underexpanded, and can traverse a highly curved trajectory. The analysis of such wall jet problems is quite formidable from both a computational and turbulence modeling viewpoint. The computation involves the analysis of a mixed subsonic/supersonic strongly interactive flowfield in a curvilinear coordinate system. Turbulence modeling considerations involve both free jet and boundary-layer concepts and must address both curvature and compressibility effects on the turbulent structure.

The problem addressed in this paper concerns the wall jet employed in high-speed circulation control (CC) airfoil applications. Dvorak and co-workers have developed a patched component CC airfoil model[1] whereby potential flow, integral boundary-layer, and finite difference wall jet solutions are iteratively coupled. The present wall jet component does not account for jet underexpansion effects or the influence of mixing on the normal pressure variation across the jet. This paper describes a new wall jet model that remedies these limitations and that will be incorporated into the CC airfoil model of Ref. 1 in the near future. A brief overview is provided.

Models Developed

To date, a pressure split, curved wall jet model (SPLITWJET[2]) and a shock-capturing curved wall jet model (SCIPWJET[3]) have been developed. SPLITWJET integrates the subsonic, turbulent wall jet equations in surface-oriented coordinates (Fig. 1). All curvature terms in the mean flow and turbulence model equations are retained, consistent with a thin-layer approximation and streamwise parabolization. A two-layer turbulence model is utilized comprised of a near-wall mixing length formulation and an outer, high Reynolds number, two-equation $k\epsilon$ model. Curvature correction terms for the $k\epsilon$ model described in Refs. 4 and 5 are incorporated. SCIPWJET predicts the multiple-cell shock structure in underexpanded wall jets flowing over curved surfaces. The governing equations are cast in mapped, surface-oriented coordinates and spatially integrated using a conservative shock-capturing algorithm. The features and capabilities of SPLITWJET and SCIPWJET are summarized in the full paper,[11] with details provided in Refs. 2 and 3.

SPLITWJET Model

SPLITWJET integrates the system of equations described by

$$\rho U\frac{\partial f}{\partial \xi}+b\left(h\rho V-\frac{a}{b}\rho U\right)\frac{\partial f}{\partial \eta}-b^2\frac{\partial}{\partial \eta}\left[\frac{h\mu_{\text{eff}}}{\sigma_f}\frac{\partial f}{\partial \eta}\right]+g_f \tag{1}$$

The equations are cast in mapped, surface-oriented coordinates (see Fig. 1) and stretching is utilized to resolve the details of the near-wall region. The f vector array contains the mean flow (velocity, total enthalpy, species) and the variables dependent upon the turbulence model; the source term g_f contains the pressure gradient and curvature related terms; μ_{eff} and σ_f are the viscosity and Prandtl numbers; a and b are mapping parameters; and $h=1-nK$, where K is the surface curvature. The boundary growth rate, $\mathrm{d}n_e/\mathrm{d}s$, is obtained via the relation

$$\frac{\mathrm{d}n_e}{\mathrm{d}s}=\frac{C_n}{U_m}\left(\frac{\partial U}{\partial \eta}\right)_e \tag{2}$$

where $\partial U/\partial \eta$ is evaluated at n_e and U_m is the maximum wall jet velocity. $C_n=1$ places the computational boundary $n_e(s)$ in close proximity with the physical edge of the wall jet δ. A fixed number of grid points spans the wall jet with a user-defined stretching parameter ($\beta=\Delta\eta_l/\Delta\eta_{l-l}$, typically equal to 1.2) employed to concentrate points in the near-wall region.

The equations are integrated with the pressure split such that the streamwise gradient $\partial P/\partial s$ is imposed in the manner proposed by Bradshaw.[6] A new, noniterative cross-flow procedure improving upon the analysis of Ref. 6 is introduced in the Appendix of Ref. 11. Applications of this new approach are given in Ref. 7.

SCIPWJET Model

SCIPWJET solves the same equations as SPLITWJET in conservation form using an explicit, shock-capturing formulation and employing the same mapping and boundary growth techniques. The present version has been utilized in the inviscid limit to analyze the wave structure associated with underexpanded wall jets. Several sample calculations are described in the full paper in Refs. 3 and 11. The viscous

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*Technical Director, Propulsion Gas Dynamics Division. Member AIAA.

†Senior Research Scientist, Propulsion Gas Dynamics Division. Member AIAA.

‡Research Scientist, Propulsion Gas Dynamics Division. Member AIAA.

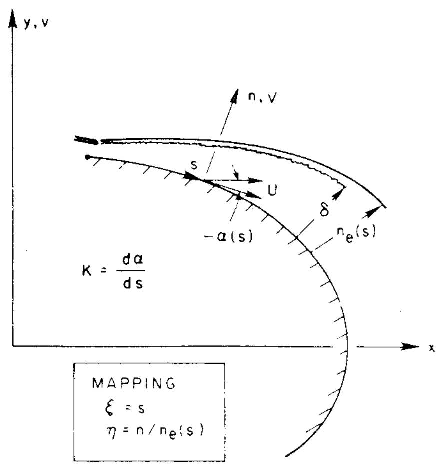

Fig. 1 Nomenclature for surface-oriented coordinate system and mapping utilized.

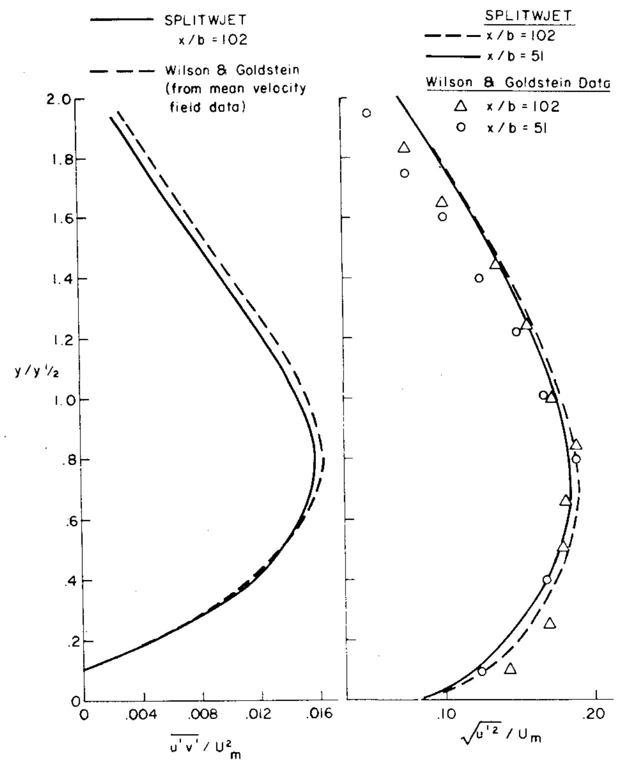

Fig. 2 Comparison of predicted turbulent stress profiles with data of Wilson and Goldstein[10] for planar subsonic wall jet with quiescent external stream.

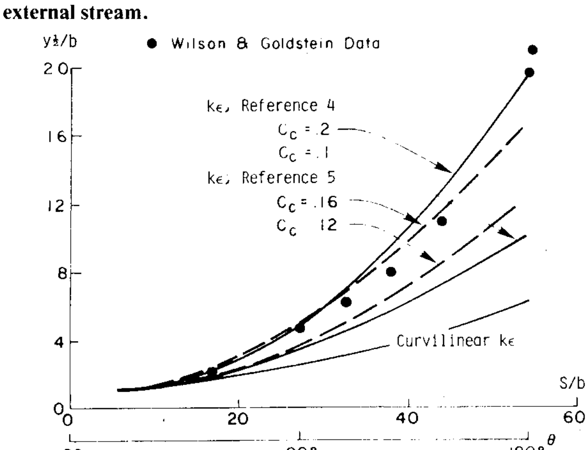

Fig. 3 Comparison of half-radius variation for subsonic wall jet on circular cylinder using standard curvilinear and curvature-corrected $k\epsilon$ turbulence models of Refs. 4 and 5.

extension is under development and will parallel the free jet modeling approach of Dash and Wolf.[8]

Comparisons with Data

Comparisons for several simple wall jet calculations are described that include wall jets exhausting into still air over planar and curved surfaces and a planar wall jet with a moving external stream. The overall agreement with the data has been quite good. For the planar, still-air calculation, the growth prediction is significantly better than that obtained by Ljuboja and Rodi[9] without requiring use of an ad hoc wall damping function (our growth rate is 0.09, theirs is 0.106, and the accepted experimental value is $dY_{1/2}/dX = 0.073$). This may be attributable to their use of a wall function approximation, since all other aspects of the modeling are comparable. Comparisons of predicted turbulent stress profiles with the data of Wilson and Goldstein[10] are exhibited in Fig. 2. The requirement for incorporating curvature-correction terms into the turbulence model can be gleaned from the growth rate comparisons (Fig. 3) for the curved wall jet case using the standard curvilinear $k\epsilon$ model and the curvature-corrected $k\epsilon$ models.[4,5] The standard model underestimates the growth rate by a factor of 4-5, while the curvature-corrected models using appropriate constants ($C_c = 0.2$ for the Ref. 4 correction, $C_c = 0.16$ for the Ref. 5 correction) do a good overall job. This is further exhibited by mean flow and turbulent stress comparisons provided in the full paper.

Acknowledgments

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